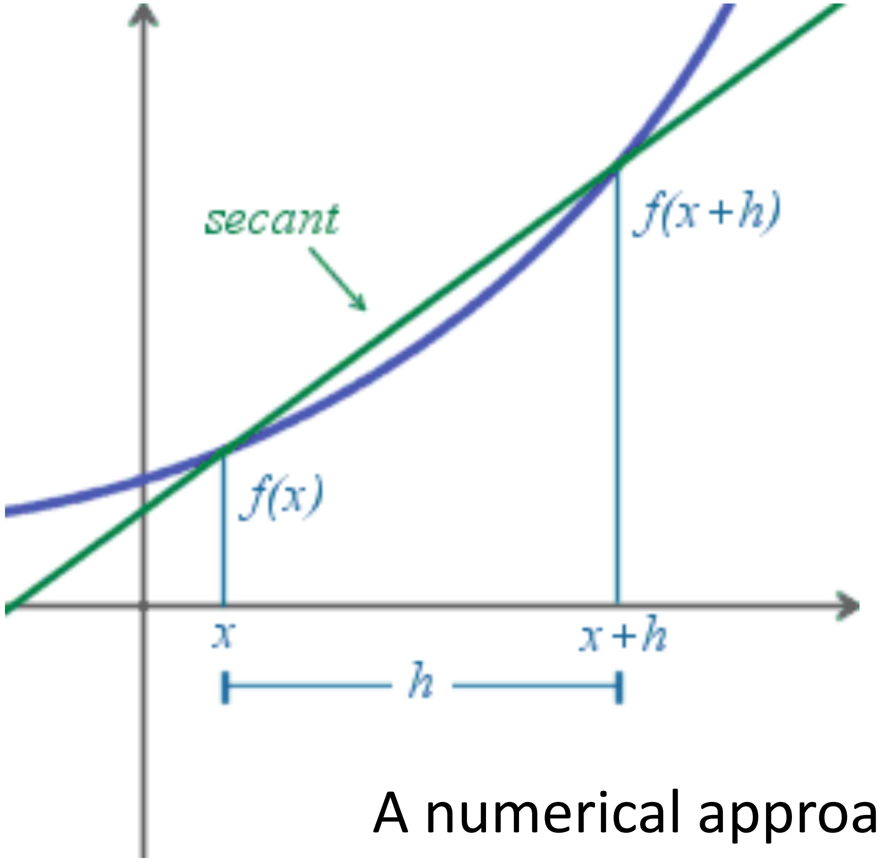
https://www.halvorsen.blog

Numerical Differentiation with MATLAB

Hans-Petter Halvorsen

Numerical Differentiation



The derivative of a function $y = f(x)$ is a measure of how y changes with x.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

A numerical approach to the derivative of a function $y = f(x)$ is:

$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note! We will use MATLAB in order to find the <u>numeric</u> solution – not the analytic solution

Numerical Differentiation

MATLAB Functions for Numerical Differentiation:

diff()

polyder()

MATLAB is a numerical language and do not perform symbolic mathematics

... well, that is not entirely true because there is “Symbolic Toolbox” available for MATLAB.

Numerical Differentiation

Given the following equation:

$$y = x^3 + 2x^2 - x + 3$$

- Find $\frac{dy}{dx}$ analytically (use “pen and paper”).
- Define a vector x from -5 to +5 and use the *diff()* function to approximate the derivative y with respect to x ($\frac{\Delta y}{\Delta x}$).
- Compare the data in a 2D array and/or plot both the exact value of $\frac{dy}{dx}$ and the approximation in the same plot.
- Increase number of data point to see if there are any difference.

Given the following equation:

$$y = x^3 + 2x^2 - x + 3$$

Then we can get the analytically solution:

$$\frac{dy}{dx} = 3x^2 + 4x - 1$$

Symbolic Math Toolbox

We start by finding the derivate of f(x) using the Symbolic Math Toolbox:

```
clear
clc

syms f(x)
syms x

f(x) = x^3 + 2*x^2 -x +3

dfdt = diff(f, x, 1)
```

This gives:

```
dfdt(x) = 3*x^2 + 4*x - 1
```

http://se.mathworks.com/help/symbolic/getting-started-with-symbolic-math-toolbox.html

```
x = -5:1:5;

% Define the function y(x)
y = x.^3 + 2*x.^2 - x + 3;

% Plot the function y(x)
plot(x,y)
title('y')

% Find nummerical solution to dy/dx
dydx_num = diff(y)./diff(x);

dydx_exact = 3*x.^2 + 4.*x -1;

dydx = [[dydx_num, NaN]', dydx_exact']

% Plot nummerical vs analytical solution to dy/dx
figure(2)
plot(x,[dydx_num, NaN], x, dydx_exact)
title('dy/dx')
legend('numerical solution', 'analytical solution')
```

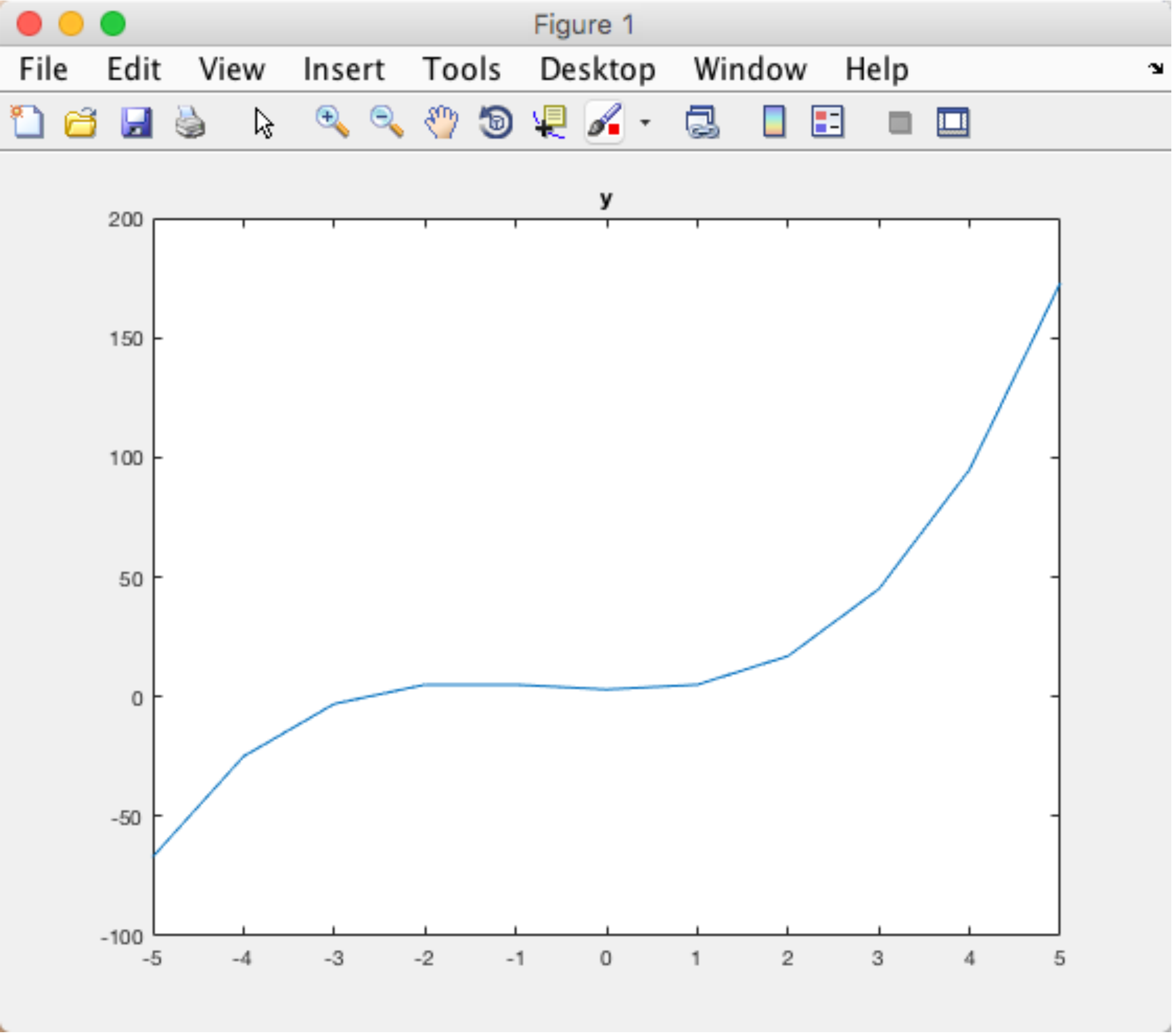
Numerical Solution

Exact Solution

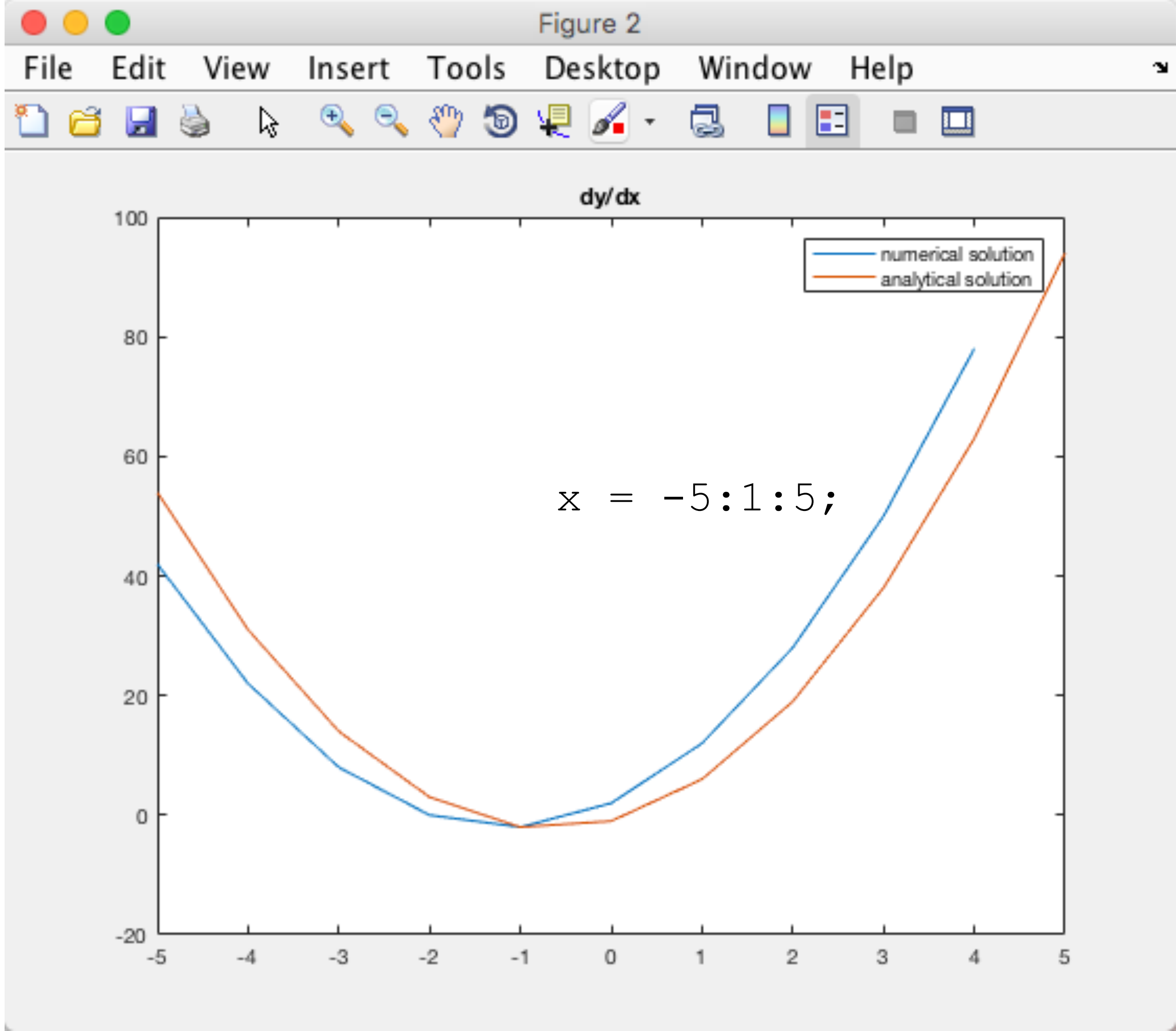
```
dydx =

    42    54
    22    31
     8    14
     0     3
    -2    -2
     2    -1
    12     6
    28    19
    50    38
    78    63
   NaN    94
```

$$y = x^3 + 2x^2 - x + 3$$

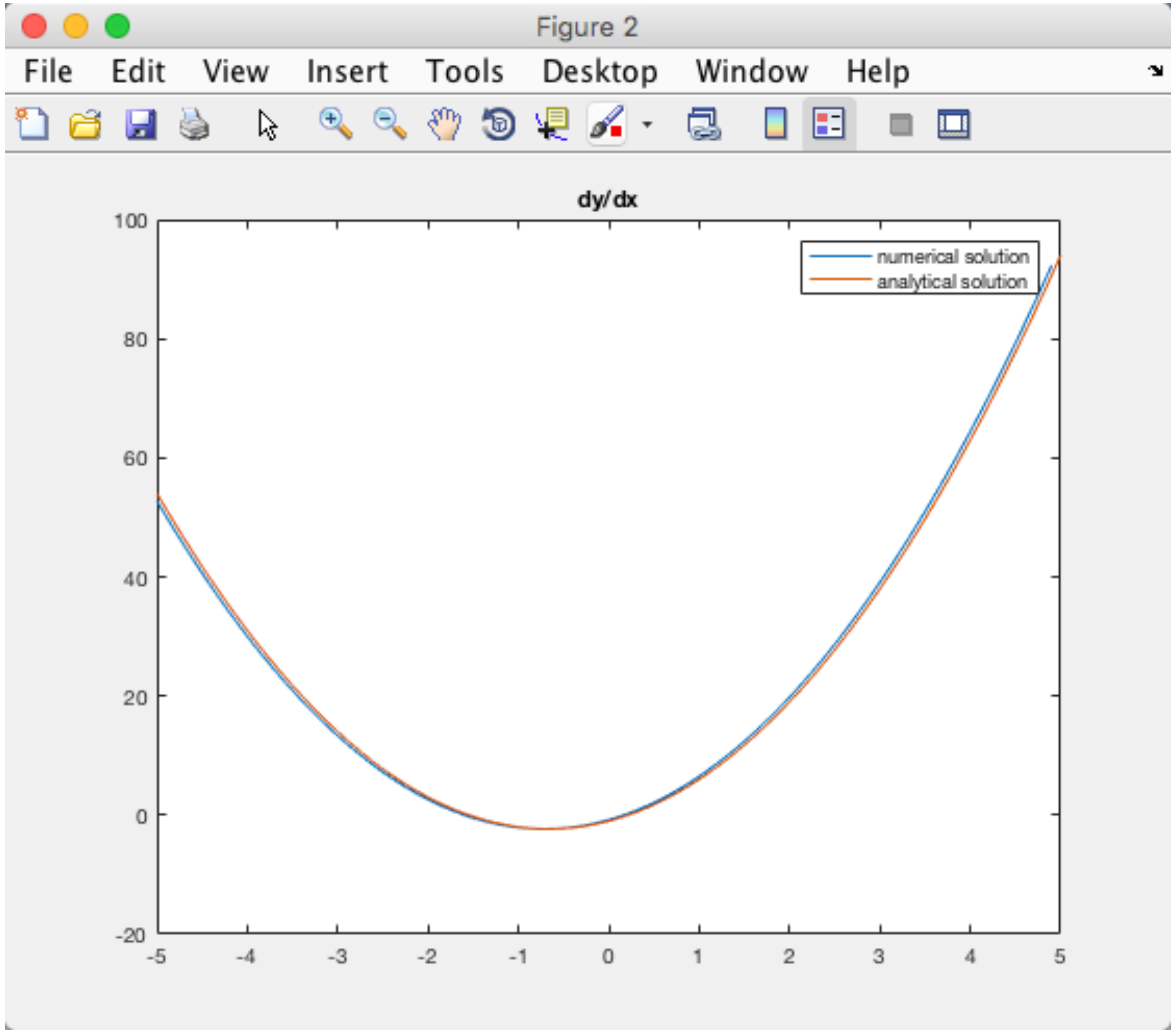


$$\frac{dy}{dx} = 3x^2 + 4x - 1$$

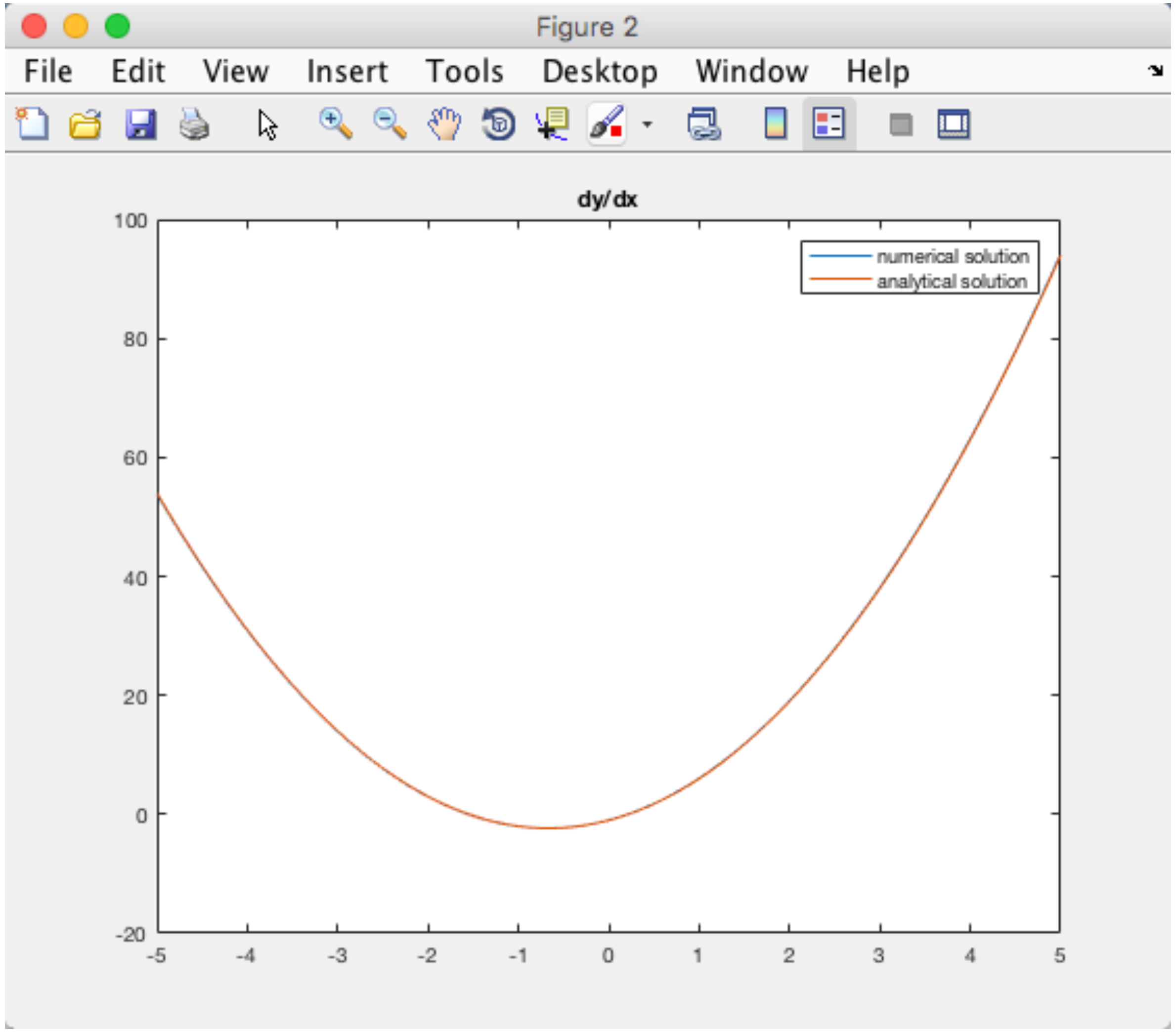


$$\frac{dy}{dx} = 3x^2 + 4x - 1$$

```
x = -5:0.1:5;
```




```
x = -5:0.01:5;
```



Differentiation on Polynomials

Given the following equation:

$$y = x^3 + 2x^2 - x + 3$$

Which is also a polynomial. A polynomial can be written on the following general form: $y(x) = a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + \ a_n$

- We will use Differentiation on the **Polynomial** to find $\frac{dy}{dx}$

From previous we know that the Analytically solution is:

$$\frac{dy}{dx} = 3x^2 + 4x - 1$$

```
p = [1 2 -1 3];

polyder(p)
```

$$y = x^3 + 2x^2 - x + 3$$

```
ans =
      3         4        -1
```

$$\frac{dy}{dx} = 3x^2 + 4x - 1$$

We see we get the correct answer

Differentiation on Polynomials

Find the derivative for the product:

$$(3x^2 + 6x + 9)(x^2 + 2x)$$

We will use the *polyder(a,b)* function.

Another approach is to use define is to first use the *conv(a,b)* function to find the total polynomial, and then use *polyder(p)* function.

Try both methods, to see if you get the same answer.

```
% Define the polynomials
p1 = [3 6 9];
p2 = [1 2 0]; %Note!

% Method 1
polyder(p1,p2)


% Method 2
p = conv(p1,p2)
polyder(p)
```

```
ans =
        12      36      42      18

p =
        3       12      21      18      0

ans =
        12      36      42      18
```

As expected, the result are the same for the 2 methods used above.

For more details, see next page.

We have that

$$p_1 = 3x^2 + 6x + 9$$

and

$$p_2 = x^2 + 2x$$

The total polynomial becomes then:

$$p = p_1 \cdot p_2 = 3x^4 + 12x^3 + 21x^2 + 18x$$

As expected, the results are the same for the 2 methods used above:

$$\frac{dp}{dx} = \frac{d(3x^4 + 12x^3 + 21x^2 + 18x)}{dx} = 12x^3 + 36x^2 + 42x + 18$$

Hans-Petter Halvorsen

University of South-Eastern Norway

www.usn.no

E-mail: hans.p.halvorsen@usn.no

Web: https://www.halvorsen.blog